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| **Technology:**  Production function: y = f(X) transforms inputs into outputs | **Cost:**  *Min W’X s.t. f(X) ≥ y0* |
| RN+  R  Typical Properties of y = f(X)   1. *Continuous*: small ΔX  small Δy 2. *Monotonic:* employing more inputs results in more (or no less) output. 3. *(Strictly) Quasi-concave on RN+*: Let xt = txo + (1-t)x’ for 0 ≤ t ≤1   f(xt) ≥= min[f(xo), f(x’)] (strict inequality for strictly quasi-concave); “upper countour sets of isoquants are convex sets”   1. *Input necessity*: f(0) = 0 | Lagrangian  *Min L (X,φ) = W’X + φ (y0 - f(X))*  *φ* is the Marginal Cost of Output  Solve ↓  **Argmin**: *X\*(W,y)* is a vector of *conditional input demands*; if a subset of inputs *X*o are fixed, *X\**(*W,y*| *X*o) is *restricted, conditional input demand*.  Plug into *W’X* Shephard’s Lemma ↓ ↑ |
| Some useful concepts  *isoquant*: Q(y) = {X ≥ 0 | f(X) = y}  *MRTSij*(X) = / defines the rate at which one input can be substituted for another while holding the level of output unchanged.  *Output Elasticity*: ωi = × =  ***Elasticity of substitution*** (σij): measure of curvature of isoquant through point X0.   * It ranges from 0 to ∞ * 0 represents perfect complements in production * ∞ represents perfect substitutes * 0 < σij < ∞ indicates two inputs that are imperfect substitutes   For homothetic *f(X)*:   * *C(W, y)* = *h(y)×C*(*W, y*=1), *h’*(*y*) >0 * *X(W, y) = g(y)×X(w*, y=1), *g’*(*y*) > 0 | LR Cost Funct: C(W, y) = *W’X\*(W,y)*  or  SR Cost Funct: C(W,y) = *W’ X\**(*W,y*|*X*o)  Properties of C(W, y):  1. C(W, y=0) = 0 in l*ong run*  (but C(W, y=0) > 0 in the *short run* due to fixed costs)   1. Continuous on its domain RN++× R+ 2. Homogeneous of degree one in input prices: 3. Non-decreasing in W 4. Concave in W 5. C() strictly increasing and unbounded in y for all W>>0, 6. Shephard’s Lemma:   The existence of a cost function, C(W, y) with properties above is consistent with a function f: RN+  R+ defined by:  f(X) = max[y≥ 0 : W’X ≥ C(W, y), W > 0]  that is increasing, unbounded above, and quasi-concave. |
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